

# Neutrino Masses and Lepton-Quark Symmetries

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The recent discovery by SuperKamiokande of evidence for neutrino masses requires the addition of at least seven new parameters to the Standard Model. We discuss the general theoretical schemes which require their inclusion, and point out how quark-lepton symmetries, either in the framework of Grand Unification, or of string theories, can be used to determine them

## 1 Neutrino Story

The Neutrino Story starts with the experiment of O. Von Bayer, O. Hahn, and L. Meitner<sup>1,2</sup> who measured the spectrum of electrons in  $\beta$  radioactivity, and found it to be discrete! In 1914, Chadwick<sup>3</sup>, then in Geiger's laboratory in Berlin came to the correct conclusion of a continuous electron spectrum, but was interned for the duration of the Great War. After much controversy, the issue was settled in 1927 by C.D. Ellis and W. A. Wooster<sup>4</sup>, who found the mean energy liberated in  $\beta$  decay accounted to be only 1/3 of the allowed energy. The stage was set for W. Pauli's famous 1930 letter.

In December of that year, in a letter that starts with typical panache, "*Dear Radioactive Ladies and Gentlemen...*", W. Pauli puts forward a "*desperate*" way out: there is a companion particle to the  $\beta$  electron. Undetected, it must be electrically neutral, and in order to balance the  $N - Li$ <sup>6</sup> statistics, it carries spin 1/2. He calls it the *neutron*, but sees no reason why it could not be massive.

In 1933, E. Fermi in his  $\beta$  decay paper gave it its final name, the little neutron or *neutrino*, as it is clearly much lighter than Chadwick's neutron which had just been discovered.

In 1945, B. Pontecorvo<sup>5</sup> proposes the unthinkable: neutrinos can be detected, through the following observation: an electron neutrino that hits a  $^{37}\text{Cl}$  atom will transform it into the inert radioactive gas  $^{37}\text{Ar}$ , which then can be stored and be detected through its radioactive decay. Pontecorvo did not publish the report, perhaps because of its secret classification, or because Fermi thought the idea ingenious but not immediately achievable. In 1954, Davis<sup>6</sup> follows up on Pontecorvo's original proposal, by setting a tank of cleaning fluid

outside a nuclear reactor.

In 1956, using a scintillation counter experiment they had proposed three years earlier<sup>7</sup>, Cowan and Reines<sup>8</sup> discover electron antineutrinos through the reaction  $\bar{\nu}_e + p \rightarrow e^+ + n$ . Cowan passed away before 1995, the year Fred Reines was awarded the Nobel Prize for their discovery. There emerge two lessons in neutrino physics: not only is patience required but also longevity: it took 26 years from birth to detection and then another 39 for the Nobel Committee to recognize the achievement! This should encourage future physicists to train their children at the earliest age to follow their footsteps, in order to establish dynasties of neutrino physicists.

In 1956, it was rumored that Davis had found evidence for neutrinos coming from a pile, and Pontecorvo<sup>9</sup>, influenced by the recent work of Gell-Mann and Pais, theorized that an antineutrino produced in the Savannah reactor could oscillate into a neutrino and be detected by Davis. The rumor went away, but the idea of neutrino oscillations was born; it has remained with us ever since, and proven the most potent tool in hunting for neutrino masses.

Having detected neutrinos, there remained to determine its spin and mass. Its helicity was measured in 1958 by M. Goldhaber<sup>10</sup>, but convincing evidence for its mass had, until SuperK's bombshell, eluded experimentalists.

After the 1957 Lee and Yang proposal of parity violation, the neutrino is again at the center of the action. Unlike the charged elementary particles which have both left- and right-handed components, weakly interacting neutrinos are purely left-handed (a ntineutrinos are right-handed), which means that lepton-number is chiral.

The second neutrino, the muon neutrino is detected<sup>11</sup> in 1962, (long anticipated by theorists Inouë and Sakata in 1943<sup>12</sup>). This time things went a bit faster as it took only 19 years from theory (1943) to discovery (1962) and 26 years to Nobel recognition (1988).

That same year, Maki, Nakagawa and Sakata<sup>13</sup> introduce two crucial ideas; one is that these two neutrinos can mix, and the second is that this mixing can cause one type of neutrino to oscillate into the other (called today flavor oscillation). This is possible only if the two neutrino flavors have different masses.

In 1963, the Astrophysics group at Caltech, Bahcall, Fowler, Iben and Sears<sup>14</sup> puts forward the most accurate of neutrino fluxes from the Sun. Their calculations included the all important Boron decay spectrum, which produces neutrinos with the right energy range for the Chlorine experiment.

In 1964, using Bahcall's result<sup>15</sup> of an enhanced capture rate of  ${}^8B$  neutrinos through an excited state of  ${}^{37}Ar$ , Davis<sup>16</sup> proposes to search for  ${}^8B$  solar neutrinos using a 100,000 gallon tank of cleaning fluid deep underground. Soon

after, R. Davis starts his epochal experiment at the Homestake mine, marking the beginning of the solar neutrino watch which continues to this day. In 1968, Davis et al reported<sup>17</sup> a deficit in the solar neutrino flux, a result that has withstood scrutiny to this day, and stands as a truly remarkable experimental *tour de force*. Shortly after, Gribov and Pontecorvo<sup>18</sup> interpreted the deficit as evidence for neutrino oscillations.

In the early 1970's, with the idea of quark-lepton symmetries<sup>19,20</sup> comes the idea that the proton could be unstable. This brings about the construction of underground (to avoid contamination from cosmic ray by-product) detectors, large enough to monitor many protons, and instrumentalized to detect the Čerenkov light emitted by its decay products. By the middle 1980's, several such detectors are in place. They fail to detect proton decay, but in a serendipitous turn of events, 150,000 years earlier, a supernova erupted in the large Magellanic Cloud, and in 1987, its burst of neutrinos was detected in these detectors! All of a sudden, proton decay detectors turn their attention to neutrinos, and to this day still waiting for its protons to decay!

As we all know, these detectors routinely monitor neutrinos from the Sun, as well as neutrinos produced by cosmic ray collisions.

## 2 Standard Model Neutrinos

The standard model of electro-weak and strong interactions contains three left-handed neutrinos. The three neutrinos are represented by two-components Weyl spinors,  $\nu_i$ ,  $i = e, \mu, \tau$ , each describing a left-handed fermion (right-handed antiferion). As the upper components of weak isodoublets  $L_i$ , they have  $I_{3W} = 1/2$ , and a unit of the global  $i$ th lepton number.

These standard model neutrinos are strictly massless. The only Lorentz scalar made out of these neutrinos is the Majorana mass, of the form  $\nu_i^t \nu_j$ ; it has the quantum numbers of a weak isotriplet, with third component  $I_{3W} = 1$ , as well as two units of total lepton number. Higgs isotriplet with two units of lepton number could generate neutrino Majorana masses, but there is no such higgs in the Standard Model: there are no tree-level neutrino masses in the standard model.

Quantum corrections, however, are not limited to renormalizable couplings, and it is easy to make a weak isotriplet out of two isodoublets, yielding the  $SU(2) \times U(1)$  invariant  $L_i^t \bar{\tau} L_j \cdot H^t \bar{\tau} H$ , where  $H$  is the Higgs doublet. As this term is not invariant under lepton number, it is not generated in perturbation theory. Thus the important conclusion: *The standard model neutrinos are kept massless by global chiral lepton number symmetry.* The detection of non-zero neutrino masses is therefore *a tangible indication of physics beyond*

the standard model.

### 3 Neutrino Mass Models

Neutrinos must be extraordinarily light: experiments indicate  $m_{\nu_e} < 10$  eV,  $m_{\nu_\mu} < 170$  keV,  $m_{\nu_\tau} < 18$  MeV<sup>21</sup>, and any model of neutrino masses must explain this suppression.

We do not discuss generating neutrino masses without new fermions, by breaking lepton number through interaction of lepton number-carrying Higgs fields.

The natural way to generate neutrinos masses is to introduce for each one its electroweak singlet Dirac partner,  $\overline{N}_i$ . These appear naturally in the Grand Unified group  $SO(10)$  where they complete each family into its spinor representation. Neutrino Dirac masses stem from the couplings  $L_i \overline{N}_j H$  after electroweak breaking. Unfortunately, these Yukawa couplings yield masses which are too big, of the same order of magnitude as the masses of the charged elementary particles  $m \sim \Delta I_w = 1/2$ .

The situation is remedied by introducing Majorana mass terms  $\overline{N}_i \overline{N}_j$  for the right-handed neutrinos. The masses of these new degrees of freedom are arbitrary, as they have no electroweak quantum numbers,  $M \sim \Delta I_w = 0$ . If they are much larger than the electroweak scale, the neutrino masses are suppressed relative to that of their charged counterparts by the ratio of the electroweak scale to that new scale: the mass matrix (in  $3 \times 3$  block form) is

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}, \quad (1)$$

leading, for each family, to one small and one large eigenvalue

$$m_\nu \sim m \cdot \frac{m}{M} \sim \left( \Delta I_w = \frac{1}{2} \right) \cdot \left( \frac{\Delta I_w = \frac{1}{2}}{\Delta I_w = 0} \right). \quad (2)$$

This seesaw mechanism<sup>22</sup> provides a natural explanation for small neutrino masses as long as lepton number is broken at a large scale  $M$ . With  $M$  around the energy at which the gauge couplings unify, this yields neutrino masses at or below tenths of eVs, consistent with the SuperK results.

The lepton flavor mixing comes from the diagonalization of the charged lepton Yukawa couplings, and of the neutrino mass matrix. From the charged lepton Yukawas, we obtain  $\mathcal{U}_e$ , the unitary matrix that rotates the lepton doublets  $L_i$ . From the neutrino Majorana matrix, we obtain  $\mathcal{U}_\nu$ , the matrix that diagonalizes the Majorana mass matrix. The  $6 \times 6$  seesaw Majorana

matrix can be written in  $3 \times 3$  block form

$$\mathcal{M} = \mathcal{V}_\nu^t \mathcal{D} \mathcal{V}_\nu \sim \begin{pmatrix} \mathcal{U}_{\nu\nu} & \epsilon \mathcal{U}_{\nu N} \\ \epsilon \mathcal{U}_{N\nu}^t & \mathcal{U}_{NN} \end{pmatrix}, \quad (3)$$

where  $\epsilon$  is the tiny ratio of the electroweak to lepton number violating scales, and  $\mathcal{D} = \text{diag}(\epsilon^2 \mathcal{D}_\nu, \mathcal{D}_N)$ , is a diagonal matrix.  $\mathcal{D}_\nu$  contains the three neutrino masses, and  $\epsilon^2$  is the seesaw suppression. The weak charged current is then given by

$$j_\mu^+ = e_i^\dagger \sigma_\mu \mathcal{U}_{MNS}^{ij} \nu_j, \quad (4)$$

where

$$\mathcal{U}_{MNS} = \mathcal{U}_e \mathcal{U}_\nu^\dagger, \quad (5)$$

is the Maki-Nakagawa-Sakata<sup>13</sup> (MNS) flavor mixing matrix, the analog of the CKM matrix in the quark sector.

In the seesaw-augmented standard model, this mixing matrix is totally arbitrary. It contains, as does the CKM matrix, three rotation angles, and one CP-violating phase. In the seesaw scenario, it also contains two additional CP-violating phases which cannot be absorbed in a redefinition of the neutrino fields, because of their Majorana masses (these extra phases can be measured only in  $\Delta\mathcal{L} = 2$  processes). These additional parameters of the seesaw-augmented standard model, need to be determined by experiment.

## 4 Theories

Theoretical predictions of lepton hierarchies and mixings depend very much on hitherto untested theoretical assumptions. In the quark sector, where the bulk of the experimental data resides, the theoretical origin of quark hierarchies and mixings is a mystery, although there exists many theories, but none so convincing as to offer a definitive answer to the community's satisfaction. It is therefore no surprise that there are more theories of lepton masses and mixings than there are parameters to be measured. Nevertheless, one can present the issues as questions:

- Do the right handed neutrinos have quantum numbers beyond the standard model?
- Are quarks and leptons related by grand unified theories?
- Are quarks and leptons related by anomalies?
- Are there family symmetries for quarks and leptons?

The measured numerical value of the neutrino mass difference (barring any fortuitous degeneracies), suggests through the seesaw mechanism, a mass for the right-handed neutrinos that is consistent with the scale at which the gauge couplings unify. Is this just a numerical coincidence, or should we view this as a hint for grand unification?

Grand unified Theories, originally proposed as a way to treat leptons and quarks on the same footing, imply symmetries much larger than the standard model's. Implementation of these ideas necessitates a desert and supersymmetry, but also a carefully designed contingent of Higgs particles to achieve the desired symmetry breaking. That such models can be built is perhaps more of a testimony to the cleverness of theorists rather than of Nature's. Indeed with the advent of string theory, we know that the best features of grand unified theories can be preserved, as most of the symmetry breaking is achieved by geometric compactification from higher dimensions<sup>23</sup>.

An alternative point of view is that the vanishing of chiral anomalies is necessary for consistent theories, and their cancellation is most easily achieved by assembling matter in representations of anomaly-free groups. Perhaps anomaly cancellation is more important than group structure.

Below, we present two theoretical frameworks of our work, in which one deduces the lepton mixing parameters and masses. One is ancient<sup>24</sup>, uses the standard techniques of grand unification, but it had the virtue of *predicting* the large  $\nu_\mu - \nu_\tau$  mixing observed by SuperKamiokande. The other<sup>25</sup> is more recent, and uses extra Abelian family symmetries to explain both quark and lepton hierarchies. It also predicts large  $\nu_\mu - \nu_\tau$  mixing. Both schemes imply small  $\nu_e - \nu_\mu$  mixings.

#### 4.1 A Grand Unified Model

The seesaw mechanism was born in the context of the grand unified group  $SO(10)$ , which naturally contains electroweak neutral right-handed neutrinos. Each standard model family is contained in two irreducible representations of  $SU(5)$ . However, the predictions of this theory for Yukawa couplings is not so clear cut, and to reproduce the known quark and charged lepton hierarchies, a special but simple set of Higgs particles had to be included. In the simple scheme proposed by Georgi and Jarlskog<sup>26</sup>, the ratios between the charged leptons and quark masses is reproduced, albeit not naturally since two Yukawa couplings, not fixed by group theory, had to be set equal. This motivated us to generalize their scheme to  $SO(10)$ , where their scheme was (technically) natural, which meant that we had an automatic window into neutrino masses through the seesaw. The Yukawa couplings were of the form

$$[A\mathbf{16}_1 \cdot \mathbf{16}_2 + B\mathbf{16}_3\mathbf{16}_3] \cdot \mathbf{126}_1 + [a\mathbf{16}_1 \cdot \mathbf{16}_2 + b\mathbf{16}_3\mathbf{16}_3] \cdot (\mathbf{10}_1 + i\mathbf{10}_2) \quad (6)$$

$$+ c\mathbf{16}_2\mathbf{16}_2 \cdot \overline{\mathbf{126}}_2 + d\mathbf{16}_2\mathbf{16}_3\overline{\mathbf{126}}_3 . \quad (7)$$

This is of course Higgs-heavy, but the attitude at the time was “damn the Higgs torpedoes, and see what happens”. This assignment was “technically” natural, enforced by two discrete symmetries. A modern treatment would include non-renormalizable operators<sup>27</sup>, rather than introducing the  $\mathbf{126}$  representations, which spoil asymptotic freedom.

The Higgs vacuum values produced the resultant masses

$$m_b = m_\tau ; \quad m_d m_s = m_e m_\mu ; \quad m_d - m_s = 3(m_e - m_\mu) . \quad (8)$$

and mixing angles

$$V_{us} = \tan \theta_c = \sqrt{\frac{m_d}{m_s}} ; \quad V_{cb} = \sqrt{\frac{m_c}{m_t}} . \quad (9)$$

While reproducing the well-known lepton and quark mass hierarchies, it predicted a long-lived  $b$  quark, contrary to the lore of the time. It also made predictions in the lepton sector, namely **maximal**  $\nu_\tau - \nu_\mu$  mixing, small  $\nu_e - \nu_\mu$  mixing of the order of  $(m_e/m_\mu)^{1/2}$ , and no  $\nu_e - \nu_\tau$  mixing.

The neutral lepton masses came out to be hierarchical, but heavily dependent on the masses of the right-handed neutrinos. The electron neutrino mass came out much lighter than those of  $\nu_\mu$  and  $\nu_\tau$ . Their numerical values depended on the top quark mass, which was then supposed to be in the tens of GeVs!

Given the present knowledge, some of the features are remarkable, such as the long-lived  $b$  quark and the maximal  $\nu_\tau - \nu_\mu$  mixing. On the other hand, the actual numerical value of the  $b$  lifetime was off a bit, and the  $\nu_e - \nu_\mu$  mixing was too large to reproduce the small angle MSW solution of the solar neutrino problem.

The lesson should be that the simplest  $SO(10)$  model that fits the observed quark and charged lepton hierarchies, reproduces, at least qualitatively, the maximal mixing found by SuperK, and predicts small mixing with the electron neutrino<sup>28</sup>.

#### 4.2 A Non-grand-unified Model

There is another way to generate hierarchies, based on adding extra family symmetries to the standard model, without invoking grand unification. These

types of models address only the Cabibbo suppression of the Yukawa couplings, and are not as predictive as specific grand unified models. Still, they predict no Cabibbo suppression between the muon and tau neutrinos. Below, we present a pre-SuperK model<sup>25</sup> with those features.

The Cabibbo suppression is assumed to be an indication of extra family symmetries in the standard model. The idea is that any standard model-invariant operator, such as  $\mathbf{Q}_i \bar{\mathbf{d}}_j H_d$ , cannot be present at tree-level if there are additional symmetries under which the operator is not invariant. Simplest is to assume an Abelian symmetry, with an electroweak singlet field  $\theta$ , as its order parameter. Then the interaction

$$\mathbf{Q}_i \bar{\mathbf{d}}_j H_d \left( \frac{\theta}{M} \right)^{n_{ij}} \quad (10)$$

can appear in the potential as long as the family charges balance under the new symmetry. As  $\theta$  acquires a *vev*, this leads to a suppression of the Yukawa couplings of the order of  $\lambda^{n_{ij}}$  for each matrix element, with  $\lambda = \theta/M$  identified with the Cabibbo angle, and  $M$  is the natural cut-off of the effective low energy theory. As a consequence of the charge balance equation

$$X_{if}^{[d]} + n_{ij} X_\theta = 0, \quad (11)$$

the exponents of the suppression are related to the charge of the standard model-invariant operator<sup>29</sup>, the sum of the charges of the fields that make up the the invariant.

This simple Ansatz, together with the seesaw mechanism, implies that the family structure of the neutrino mass matrix is determined by the charges of the left-handed lepton doublet fields.

Each charged lepton Yukawa coupling  $L_i \bar{N}_j H_u$ , has an extra charge  $X_{L_i} + X_{N_j} + X_H$ , which gives the Cabibbo suppression of the  $ij$  matrix element. Hence, the orders of magnitude of these couplings can be expressed as

$$\begin{pmatrix} \lambda^{l_1} & 0 & 0 \\ 0 & \lambda^{l_2} & 0 \\ 0 & 0 & \lambda^{l_3} \end{pmatrix} \hat{Y} \begin{pmatrix} \lambda^{p_1} & 0 & 0 \\ 0 & \lambda^{p_2} & 0 \\ 0 & 0 & \lambda^{p_3} \end{pmatrix}, \quad (12)$$

where  $\hat{Y}$  is a Yukawa matrix with no Cabibbo suppressions,  $l_i = X_{L_i}/X_\theta$  are the charges of the left-handed doublets, and  $p_i = X_{N_i}/X_\theta$ , those of the singlets. The first matrix forms half of the MNS matrix. Similarly, the mass matrix for the right-handed neutrinos,  $\bar{N}_i \bar{N}_j$  will be written in the form

$$\begin{pmatrix} \lambda^{p_1} & 0 & 0 \\ 0 & \lambda^{p_2} & 0 \\ 0 & 0 & \lambda^{p_3} \end{pmatrix} \mathcal{M} \begin{pmatrix} \lambda^{p_1} & 0 & 0 \\ 0 & \lambda^{p_2} & 0 \\ 0 & 0 & \lambda^{p_3} \end{pmatrix}. \quad (13)$$



The diagonalization of the seesaw matrix is of the form

$$L_i H_u \overline{N}_j \left( \frac{1}{\overline{N} \overline{N}} \right)_{jk} \overline{N}_k H_u L_l , \quad (14)$$

from which the Cabibbo suppression matrix from the  $\overline{N}_i$  fields *cancels*, leaving us with

$$\begin{pmatrix} \lambda^{l_1} & 0 & 0 \\ 0 & \lambda^{l_2} & 0 \\ 0 & 0 & \lambda^{l_3} \end{pmatrix} \hat{\mathcal{M}} \begin{pmatrix} \lambda^{l_1} & 0 & 0 \\ 0 & \lambda^{l_2} & 0 \\ 0 & 0 & \lambda^{l_3} \end{pmatrix} , \quad (15)$$

where  $\hat{\mathcal{M}}$  is a matrix with no Cabibbo suppressions. The Cabibbo structure of the seesaw neutrino matrix is determined solely by the charges of the lepton doublets! As a result, the Cabibbo structure of the MNS mixing matrix is also due entirely to the charges of the three lepton doublets. This general conclusion depends on the existence of at least one Abelian family symmetry, which we argue is implied by the observed structure in the quark sector.

The Wolfenstein parametrization of the CKM matrix<sup>30</sup>,

$$\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} , \quad (16)$$

and the Cabibbo structure of the quark mass ratios

$$\frac{m_u}{m_t} \sim \lambda^8 \quad \frac{m_c}{m_t} \sim \lambda^4 \quad ; \quad \frac{m_d}{m_b} \sim \lambda^4 \quad \frac{m_s}{m_b} \sim \lambda^2 , \quad (17)$$

can be reproduced<sup>25,31</sup> by a simple *family-traceless* charge assignment for the three quark families, namely

$$X_{\mathbf{Q}, \overline{\mathbf{u}}, \overline{\mathbf{d}}} = \mathcal{B}(2, -1, -1) + \eta_{\mathbf{Q}, \overline{\mathbf{u}}, \overline{\mathbf{d}}}(1, 0, -1) , \quad (18)$$

where  $\mathcal{B}$  is baryon number,  $\eta_{\overline{\mathbf{d}}} = 0$ , and  $\eta_{\mathbf{Q}} = \eta_{\overline{\mathbf{u}}} = 2$ . Two striking facts are evident:

- the charges of the down quarks,  $\overline{\mathbf{d}}$ , associated with the second and third families are the same,
- $\mathbf{Q}$  and  $\overline{\mathbf{u}}$  have the same value for  $\eta$ .

To relate these quark charge assignments to those of the leptons, we need to inject some more theoretical prejudices. Assume these family-traceless charges

are gauged, and not anomalous. Then to cancel anomalies, the leptons must themselves have family charges.

Anomaly cancellation generically implies group structure. In  $SO(10)$ , baryon number generalizes to  $\mathcal{B} - \mathcal{L}$ , where  $\mathcal{L}$  is total lepton number, and in  $SU(5)$  the fermion assignment is  $\mathbf{\bar{5}} = \mathbf{\bar{d}} + L$ , and  $\mathbf{10} = \mathbf{Q} + \mathbf{\bar{u}} + \mathbf{\bar{e}}$ . Thus anomaly cancellation is easily achieved by assigning  $\eta = 0$  to the lepton doublet  $L_i$ , and  $\eta = 2$  to the electron singlet  $\bar{e}_i$ , and by generalizing baryon number to  $\mathcal{B} - \mathcal{L}$ , leading to the charges

$$X_{\mathbf{Q}, \mathbf{\bar{u}}, \mathbf{\bar{d}}, L, \bar{e}} = (\mathcal{B} - \mathcal{L})(2, -1, -1) + \eta_{\mathbf{Q}, \mathbf{\bar{u}}, \mathbf{\bar{d}}}(1, 0, -1) , \quad (19)$$

where now  $\eta_{\mathbf{\bar{d}}} = \eta_L = 0$ , and  $\eta_{\mathbf{Q}} = \eta_{\mathbf{\bar{u}}} = \eta_{\bar{e}} = 2$ . It is interesting to note that  $\eta$  is at least in  $E_6$ . The origin of such charges is not clear, as it implies in the superstring context, rather unconventional compactification.

As a result, the charges of the lepton doublets are simply  $X_{L_i} = -(2, -1, -1)$ . We have just argued that these charges determine the Cabibbo structure of the MNS lepton mixing matrix to be

$$\mathcal{U}_{MNS} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{pmatrix} , \quad (20)$$

implying *no Cabibbo suppression in the mixing between  $\nu_\mu$  and  $\nu_\tau$* . This is consistent with the SuperK discovery and with the small angle MSW<sup>33</sup> solution to the solar neutrino deficit. One also obtains a much lighter electron neutrino, and Cabibbo-comparable masses for the muon and tau neutrinos. Notice that these predictions are subtly different from those of grand unification, as they yield  $\nu_e - \nu_\tau$  mixing. It also implies a much lighter electron neutrino, and Cabibbo-comparable masses for the muon and tau neutrinos.

On the other hand, the scale of the neutrino mass values depend on the family trace of the family charge(s). Here we simply quote the results our model<sup>25</sup>. The masses of the right-handed neutrinos are found to be of the following orders of magnitude

$$m_{\bar{N}_e} \sim M\lambda^{13} ; \quad m_{\bar{N}_\mu} \sim m_{\bar{N}_\tau} \sim M\lambda^7 , \quad (21)$$

where  $M$  is the scale of the right-handed neutrino mass terms, assumed to be the cut-off. The seesaw mass matrix for the three light neutrinos comes out to be

$$m_0 \begin{pmatrix} a\lambda^6 & b\lambda^3 & c\lambda^3 \\ b\lambda^3 & d & e \\ c\lambda^3 & e & f \end{pmatrix} , \quad (22)$$

where we have added for future reference the prefactors  $a, b, c, d, e, f$ , all of order one, and

$$m_0 = \frac{v_u^2}{M\lambda^3} , \quad (23)$$

where  $v_u$  is the  $vev$  of the Higgs doublet. This matrix has one light eigenvalue

$$m_{\nu_e} \sim m_0 \lambda^6 . \quad (24)$$

Without a detailed analysis of the prefactors, the masses of the other two neutrinos come out to be both of order  $m_0$ . The mass difference announced by superK<sup>32</sup> cannot be reproduced without going beyond the model, by taking into account the prefactors. The two heavier mass eigenstates and their mixing angle are written in terms of

$$x = \frac{df - e^2}{(d + f)^2} , \quad y = \frac{d - f}{d + f} , \quad (25)$$

as

$$\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}} , \quad \sin^2 2\theta_{\mu\tau} = 1 - \frac{y^2}{1 - 4x} . \quad (26)$$

If  $4x \sim 1$ , the two heaviest neutrinos are nearly degenerate. If  $4x \ll 1$ , a condition easy to achieve if  $d$  and  $f$  have the same sign, we can obtain an adequate split between the two mass eigenstates. For illustrative purposes, when  $0.03 < x < 0.15$ , we find

$$4.4 \times 10^{-6} \leq \Delta m_{\nu_e - \nu_\mu}^2 \leq 10^{-5} \text{ } rmeV^2 , \quad (27)$$

which yields the correct non-adiabatic MSW effect, and

$$5 \times 10^{-4} \leq \Delta m_{\nu_\mu - \nu_\tau}^2 \leq 5 \times 10^{-3} \text{ } eV^2 , \quad (28)$$

for the atmospheric neutrino effect. These were calculated with a cut-off,  $10^{16} \text{ GeV} < M < 4 \times 10^{17} \text{ GeV}$ , and a mixing angle,  $0.9 < \sin^2 2\theta_{\mu-\tau} < 1$ . This value of the cut-off is compatible not only with the data but also with the gauge coupling unification scale, a necessary condition for the consistency of our model, and more generally for the basic ideas of Grand Unification.

## 5 Outlook

Exact predictions of neutrino masses and mixings depend on developing a credible theory of flavor. In the absence of such, we have presented two schemes, which predicted not only maximal  $\nu_\mu - \nu_\tau$  mixing, but also small  $\nu_e - \nu_\mu$  mixings.

Neither scheme includes sterile neutrinos. The present experimental situation is somewhat unclear: the LSND results<sup>34</sup> imply the presence of a sterile neutrino; at this conference we heard that superK favors  $\nu_\mu - \nu_\tau$  oscillation over  $\nu_\mu - \nu_{\text{sterile}}$ , and the origin of the solar neutrino deficit remains a puzzle, which several possible explanations. One is the non-adiabatic MSW effect in the Sun, which our theoretical ideas seem to favor. However, it is an experimental question which is soon to be answered by the continuing monitoring of the  $^8B$  spectrum by SuperK, and the advent of the SNO detector. Neutrino physics is at an exciting stage, and experimentally vibrant, as upcoming measurements will help us determine our basic ideas about fundamental interactions.

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